5 APPLICATION OF THE FINITE ELEMENT METHOD IN DETERMINATION OF THE NATURAL VIBRATIONS FREQUENCY OF FRICTIONAL JOINT

5.1 Introduction

During the operation the yielding support of dog headings is endangered on the action of static and dynamic loading from the side of a rock mass. Especially dangerous from the dog headings' functionality preservation and the safety of personnel point of view, is the action of dynamic loading, whose source are rock bursts, decompressions of the rock mass and shocks occurring within the rock mass [1]. Yielding support of dog headings is endangered most often on the action of impulse (impact) loadings, depending on the short duration and sudden action of external forces, and on the action of pulsatory loadings related to dynamic movements of headings' contours, as a result of vibrations of the rock mass [2, 3, 5].

Studies, whose aim was to determine the dynamic parameters of vibrations of heading's contour caused by the shocks of rock mass proved, that maximum value of displacements of dog heading's contour was occurring at the frequency of vibrations in the range of 4-10 Hz [3]. G. Mutke in [5] also stated, that frequencies of vibrations of heading's contour, at the near field of displacements, vary in the range from 20 to 300 Hz. On the basis of data presented in these papers it can be stated, that in mining practice the range of frequencies of vibrations of dog heading's contour amount from 4 Hz to 300 Hz.

The more frequent occurrence of dynamic phenomena causes the necessity of conductance the studies in order to determine the influence of these phenomena on the support.

Frictional joint, as the main part of friction prop and steel frames of yielding support of dog headings, has decisive influence on their loading capacity and yield capacity, what subsequently influences on the operational parameters of the whole support.

Therefore, it becomes purposeful to carry out the modal analysis of the frictional joint, depending on the determination of its free vibration frequency and the form of these vibrations, and then the comparison of frequencies of these vibrations, with the frequencies of vibrations of heading's contour determined during the tests. Such comparison will enable to determine the free vibration frequency of the frictional joint being within the range of possible vibrations of heading's contour. For these frequencies there exists a danger of occurrence of large displacements of joint's elements, what can lead to their damage and the loss of support's stability. Presented analysis included frictional joints made of V29 section with two and three SDO29 type stirrups. Determinations of free vibration frequencies, and their forms for the joints being investigated, were carried out with the use of finite element method.

5.2 Assumptions of modal analysis

Modal analysis is commonly applied technique of investigation of the dynamic properties of mechanical objects. As its result one get modal model in a form of set of free vibration frequencies, damping coefficients at these frequencies, and the forms of vibrations. On the basis of the modal model the behaviour of an object in the situations, in which dynamic loadings act on the mechanism being investigated, can be predicted. Method of modal analysis is applied among the others in order to diagnose the state of construction, dynamic modification of the construction's properties etc. Depending on the way of identification of the modal model, the theoretical, experimental and exploratory modal analysis can be distinguished [4, 6, 7].

In a case of investigated frictional joint the theoretical modal analysis of its structural model was conducted. The model was built with the aid of the spatial modelling techniques, using finite element method, which depends on the determination of its dynamic properties.

Modal analysis can be applied, if mechanical structure of the investigated object fulfills the following assumptions [6]:

- The linearity rule in a domain of description of the dynamics by the linear system of ordinary or partial differential equations is fulfilled,
- Constancy in time of coefficients in the equations describing the dynamics of construction,
- System is observable and there exists possibility of measurement of all characteristics, knowledge of which is necessary to identify the model,
- System fulfills the maxwell reciprocal theorem, i.e. Answer at i-th point for the I put function at k-th point is equal to the answer in k-th point for the same input function at i-th point. This assumption causes, that the matrices of mass, rigidity, damping and frequency characteristics are symmetrical.
- Damping in a system is small or proportional.

In a case of dynamic analysis of given construction of n degrees of freedom the matrix form of dynamic differential equations can be written in a form:

$$M \cdot \ddot{y} + C \cdot \dot{y} + K \cdot y = P(t) \tag{5.1}$$

where: M – inertia matrix (n x n) of model,
C – damping matrix of model,
K – rigidity matrix of model,
P(t) – generalized forces vector,
$\ddot{y}$ – generalized accelerations vector,
$\dot{y}$ – generalized velocities vector,
y – generalized displacements vector.

For the system without the damping, which is not subjected to the external loading, the equation of motion will have a form:

$$M \cdot \ddot{y} + K \cdot y = 0 \tag{5.2}$$

It is general equation of motion of undamped vibrations and it constitutes the basis of analysis of free vibrations of investigated object.

System described with equation(5.2) will freely vibrate in a harmonic form defined by the equation of a form:

$$y(t) = y^o \sin(\omega t + \varphi) \tag{5.3}$$

where: y^o – matric of amplitudes of generalized displacements,
ω – angular frequency,
φ – phase angle.

After substitution of equation (5.3) into relationship (5.2) one get the system of uniform algebraic linear equations in a function of generalized displacement (y) in a form:

$$(K - M \cdot \omega^2) \cdot y = 0 \tag{5.4}$$

The solution of equation (5.4) is determination of n roots being the real values of ω^2, which determine free vibrations of investigated system. Each determined natural frequency corresponds to the different vector of a form of vibration yn given in a scaled form, whose components keep constant mutual relation, but values of each of them can be different [4].

General scheme of construction of theoretical modal analysis is presented in a fig. 5.1.

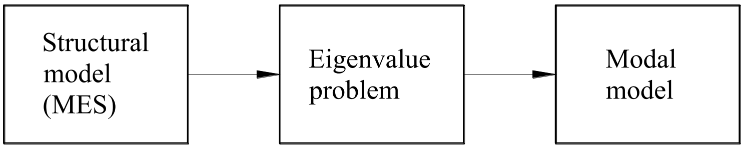


Fig. 5.1 Scheme of theoretical modal analysis

5.3 Model of frictional joint

In order to carry out the modal analysis in a domain of determination of free vibration frequency and their form, a spatial structural model of the frictional joint was developed. This model was developed basing on the real frictional joint made of V29 section with SDO29 type stirrups. Material properties were taken according to their real values, which amounted to:

- Young's modulus: E = 210 GPa,
- Poisson's ratio: ν = 0,3,
- material density: ρ = 785 kg/m^3.

In a fig. 5.2 the discrete model of frictional joint with two (fig. 5.2 a) and three stirrups (fig. 5.2 b) is presented.

a)

b)

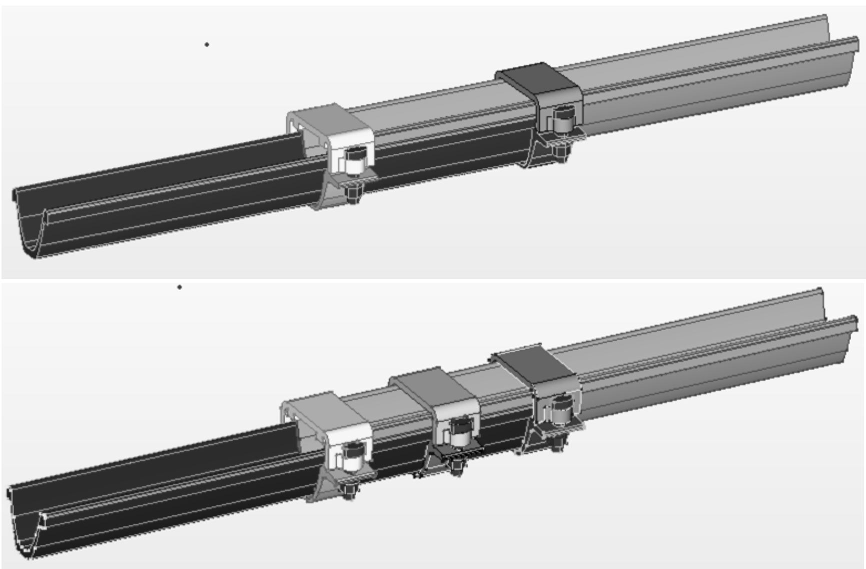


Fig. 5.2 Discrete model of frictional joint with two (a) and three (b) stirrups

5.4 Results of modal analysis

Modal analysis was carried out for frictional joints with two and three stirrups at two different ways of their attachment. In a first case one end of the joint was rigidly fixed, and in a second case the both ends were fixed. Hence, four cases were subjected to the analysis. For each of these cases the first 10 forms of free vibrations was determined.

Forms of free vibrations are presented for frequencies up to 300 Hz, assuming that it is maximum frequency of vibrations of dog heading's contour. In a tab. 5.1 free vibration frequencies for one-sided rigidly fixed frictional joint with three stirrups are listed.

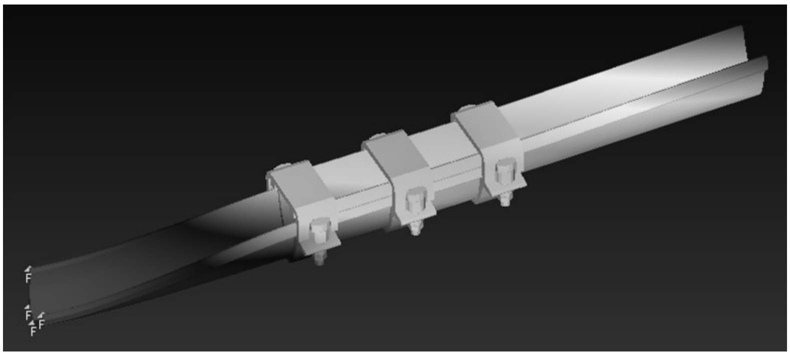
Tab. 5.1 List of free vibration frequencies of the frictional joint with three stirrups one-sided rigidly fixed

Number of form of free vibrations	**1**	**2**	**3**	**4**	**5**
Frequency, Hz	28,03	29,08	76,13	166,38	183,19
Number of form of free vibrations	**6**	**7**	**8**	**9**	**10**
Frequency, Hz	334,80	403,96	446,60	480,76	522,28

For each of calculated frequencies also forms of free vibrations were determined, of which first five are presented in fig. 5.3 and fig. 5.4.

a)

$f_1 = 28{,}03\ Hz$



b)

$f_2 = 29{,}08\ Hz$

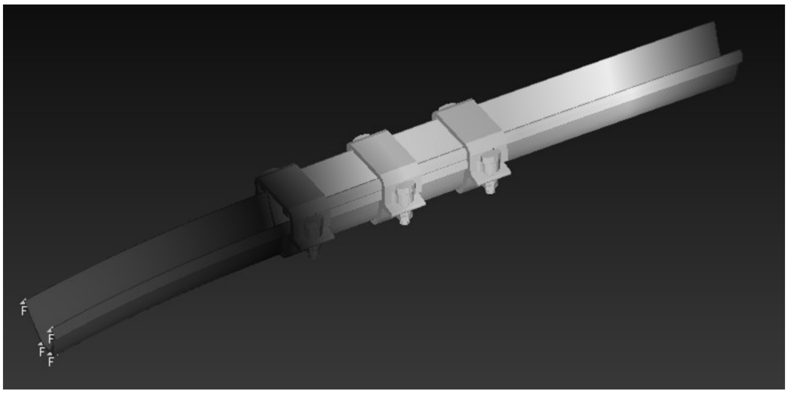


Fig. 5.3 First (a) and second (b) form of free vibration frequencies of the frictional joint with three stirrups one-sided rigidly fixed

a)

$f_4 = 166{,}38\ Hz$

b)

$f_5 = 183{,}19\ Hz$

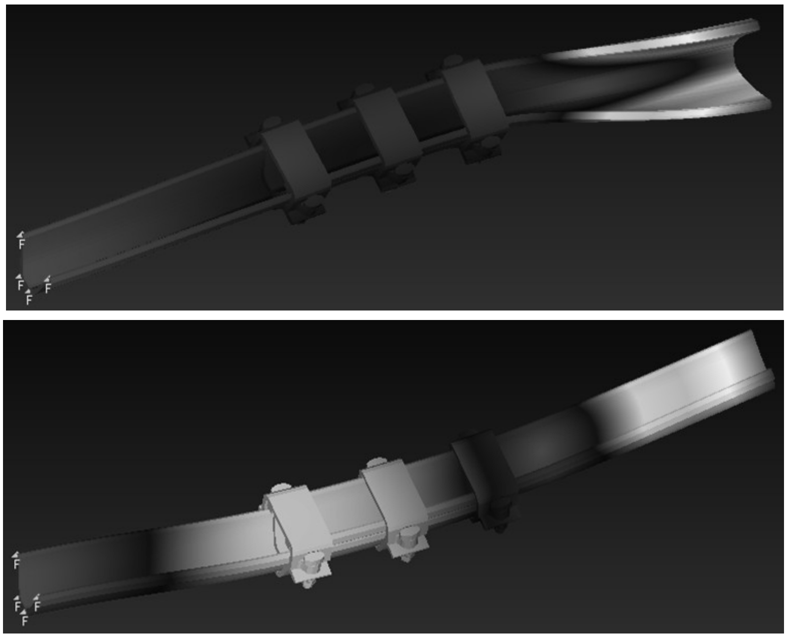


Fig. 5.4 Fourth (a) and fifth (b) form of free vibration frequencies of the frictional joint with three stirrups one-sided rigidly fixed

In a tab. 5.2 free vibration frequencies for two-sided rigidly fixed frictional joint with three stirrups are listed.

Tab. 5.2 List of free vibration frequencies for the frictional joint with three stirrups two-sided rigidly fixed

Number of form of free vibrations	**1**	**2**	**3**	**4**	**5**
Frequency, Hz	71,21	148,00	236,08	339,49	431,40
Number of form of free vibrations	**6**	**7**	**8**	**9**	**10**
Frequency, Hz	443,91	550,45	594,50	603,23	831,21

a)

$f_1 = 71{,}21\ Hz$

b)

$f_2 = 148{,}00\ Hz$

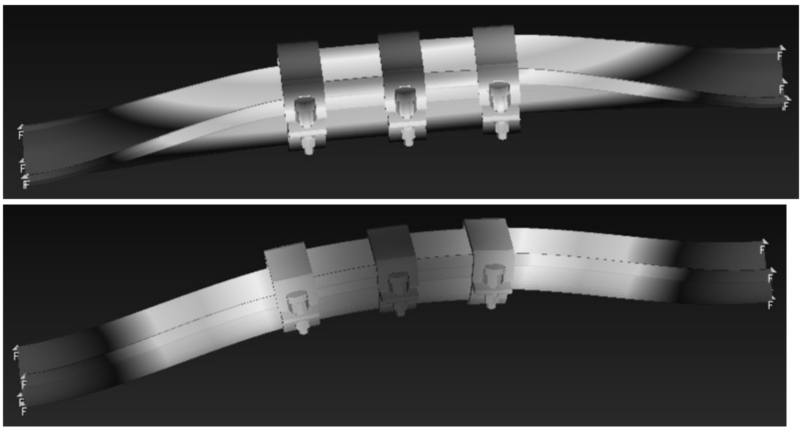


Fig. 5.5 First (a) and second (b) form of free vibration frequencies of frictional joint with three stirrups two-sided rigidly fixed

In fig. 5.5 and fig. 5.6 first four forms of free vibrations of frictional joint with three stirrups two-sided rigidly fixed.

a)

$f_3 = 236{,}08\ Hz$

b)

$f4 = 339{,}49\ Hz$

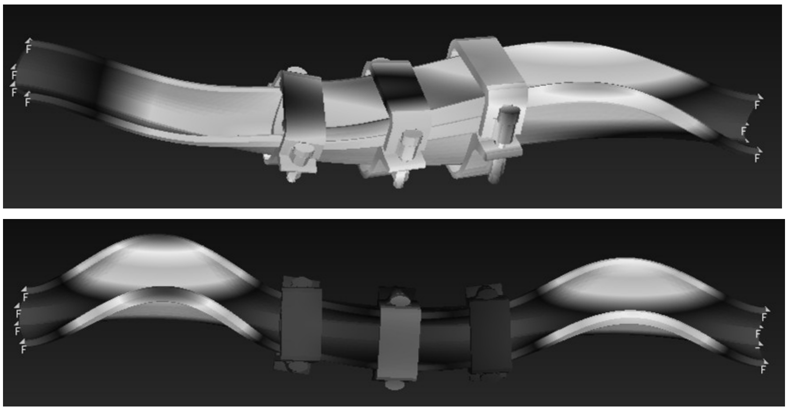


Fig. 5.6 Third (a) and fourth (b) form of free vibration frequencies of frictional joint with three stirrups two-sided rigidly fixed

In a tab. 5.3 free vibration frequencies for two-sided rigidly fixed frictional joint with two stirrups are listed.

Tab. 5.3 List of free vibration frequencies for the frictional joint with two stirrups two-sided rigidly fixed

Number of form of free vibrations	**1**	**2**	**3**	**4**	**5**
Frequency, Hz	77,57	158,38	254,38	340,85	435,49
Number of form of free vibrations	**6**	**7**	**8**	**9**	**10**
Frequency, Hz	451,85	555,42	710,69	896,33	963,44

In fig. 5.7 and fig. 5.8 first four forms of free vibrations of frictional joint with two stirrups two-sided rigidly fixed.

a)

$f_1 = 77{,}57\ Hz$

b)

$f_2 = 158{,}38\ Hz$

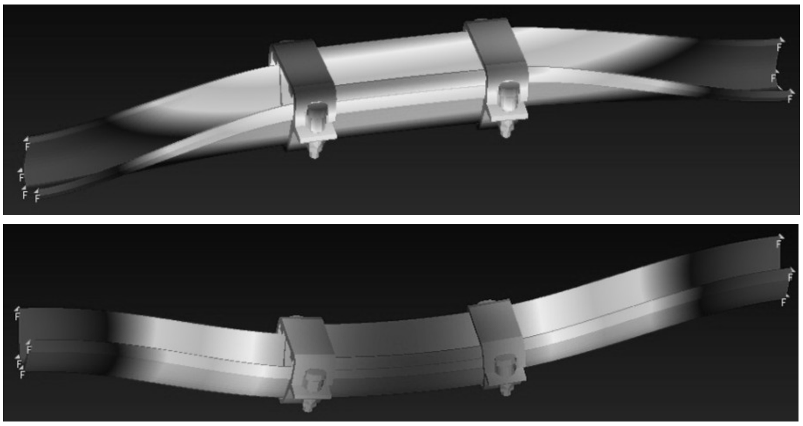


Fig. 5.7 First (a) and second (b) form of free vibration frequencies of frictional joint with two stirrups two-sided rigidly fixed

a)

$f3 = 254{,}38\ Hz$

b)

$f_4 = 340{,}85\ Hz$

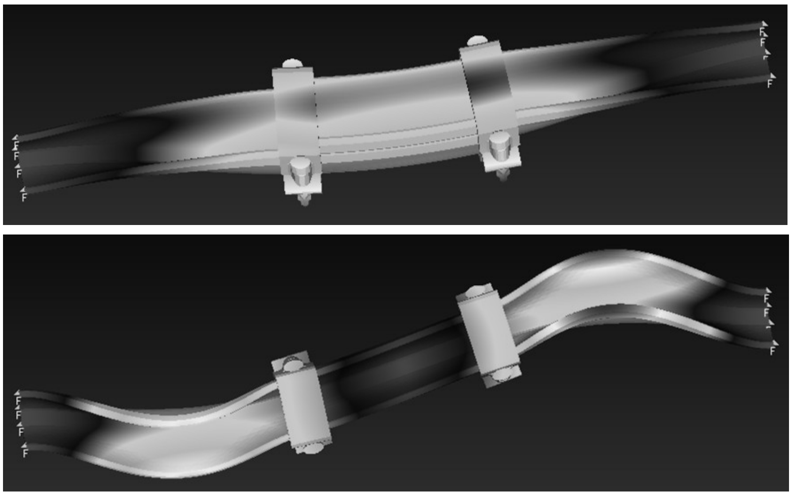


Fig. 5.8 Third (a) and fourth (b) form of free vibration frequencies of frictional joint with two stirrups two-sided rigidly fixed

In a tab. 5.4 free vibration frequencies for one-sided rigidly fixed frictional joint with two stirrups are listed.

Tab. 5.4 List of free vibration frequencies for the frictional joint with two stirrups one-sided rigidly fixed

Number of form of free vibrations	**1**	**2**	**3**	**4**	**5**
Frequency, Hz	29,14	29,92	81,30	170,96	190,58
Number of form of free vibrations	**6**	**7**	**8**	**9**	**10**
Frequency, Hz	344,32	406,91	454,88	488,23	565,37

In fig. 5.9, fig. 5.10 and fig. 5.11 first five forms of free vibrations of frictional joint with two stirrups one-sided rigidly fixed.

a)

$f_1 = 29{,}314 Hz$

b)

$f_2 = 29{,}92\ Hz$

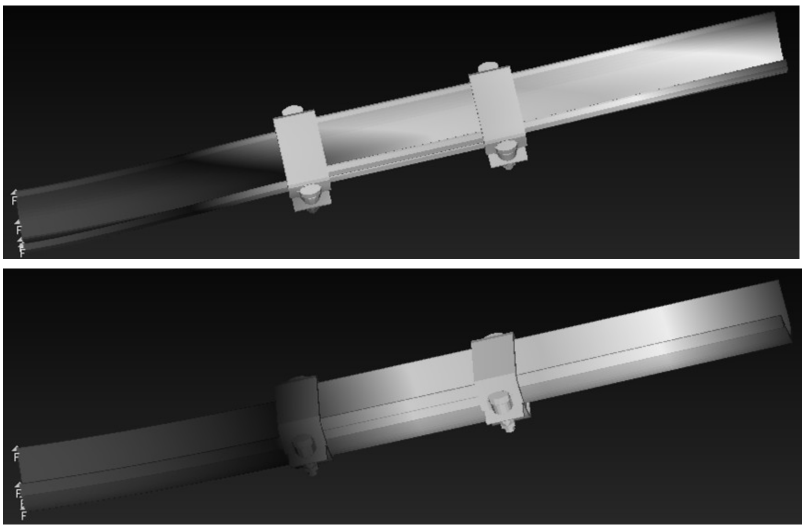
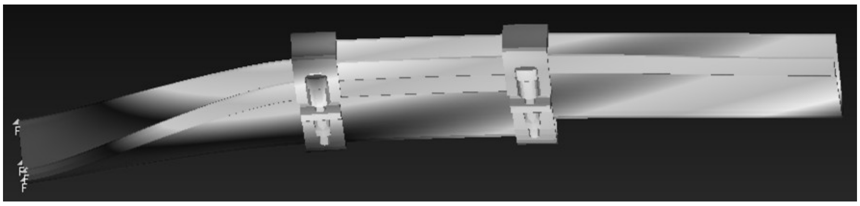


Fig. 5.9 First (a) and second (b) form of free vibration frequencies of frictional joint with two stirrups one-sided rigidly fixed

a)

$f3 = 81{,}30\ Hz$



b)

$f_4 = 170{,}96\ Hz$

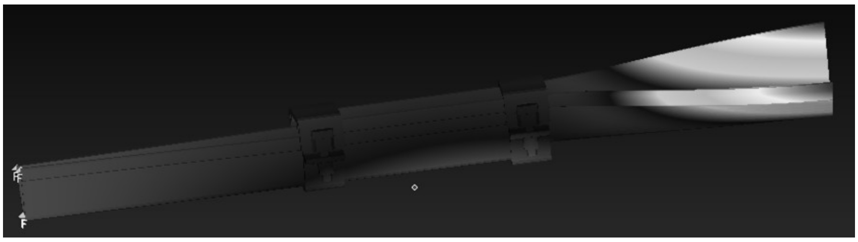


Fig. 5.10 Third (a) and fourth (b) form of free vibration frequencies of frictional joint with two stirrups one-sided rigidly fixed

$f_5 = 190{,}58\ Hz$

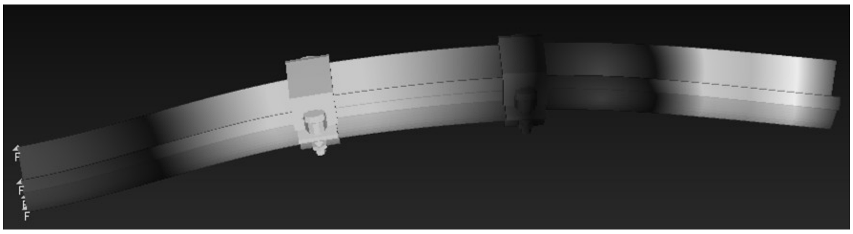


Fig. 5.11 Fifth form of free vibration frequencies of frictional joint with two stirrups one-sided rigidly fixed

5.5 Summary and Conclusions

Application of finite element method to modal analysis of frictional joint used in mining yielding support enabled the determination of its free vibration frequency and the form of these vibrations. Due to the more frequently occurring in the practice dynamic interaction of rock mass on the mining headings, determined quantities have very significant meaning for the safety of use of the yielding support.

In domain of the analysis carried out it can be stated, that very significant influence on the value of free vibration frequency has the way of attachment of the frictional joint.

Comparing determined free vibration frequencies it can be stated, that for the joint one-sided rigidly fixed their values are much lower than for joint two-sided rigidly fixed. It causes an increase of values of frequencies of mass rock's vibrations, at which dangerous resonance vibrations can occur.

Number of stirrups in the frictional joint has lesser influence on a value of free vibration frequencies. These frequencies are slightly higher for the frictional joints with two stirrups.

The lowest free vibration frequency of frictional joint was registered for the frictional joint one-sided rigidly fixed with three stirrups and it amounted to 28,03 Hz. This values is quite distant from the most dangerous range of frequency of vibrations of dog heading's contour (4-10 Hz).

Analyzing obtained forms of free vibrations it can be stated, that irrespective of the way of attachment and the number of stirrups in the joint both, sections and stirrups, are endangered by the great deformations.

On the basis of determined free vibration frequencies of frictional joints it can be stated, that their values are high and should not cause too great hazard of support damage.

It seems justified to carry out further investigations on identification of loading state of dog headings support.

Presented analysis of frequencies and forms of free vibration in frictional joint should be significant source of information for users of yielding support of dog headings, and useful help at selection of support for headings endangered by dynamic interaction of rock mass.

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