Statystyka

Wprowadzenie

$$k \leq 5 \cdot \log(N)$$

$$N = \sum_{i=1}^{N} n_i$$

$$\dot{x} = \frac{x_{od} + x_{od}}{2}$$

Statystyka opisowa

Miary położenia

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{k} x_i n_i$$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{k} \dot{x}_i n_i$$

$$D_0 = x_{(\max n_i)}$$

$$D_0 = x_0 + \frac{n_0 - n_-}{(n_0 - n_-) + (n_0 - n_+)}$$

$$Q_1 = x_0 + \left(\frac{N}{4} - n_{icum-}\right)\frac{\Delta x}{n_0}$$

$$Q_2 = x_0 + \left(\frac{N}{2} - n_{icum-}\right)\frac{\Delta x}{n_0}$$

$$Q_3 = x_0 + \left(\frac{3}{4}N - n_{icum-}\right)\frac{\Delta x}{n_0}$$

$$\bar{x} - D_0 \approx 3(\bar{x} - Me)$$

$$Q_2 = Me$$

Miary zmienności

$$S^2x = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

$$S^2x = \frac{1}{N}\sum_{i=1}^{k}(x_i - \bar{x})^2 n_i$$

$$S^2x = \frac{1}{N}\sum_{i=1}^{k}(\dot{x}_i - \bar{x})^2 n_i$$

$$Sx = \sqrt{S^2x}$$

$$\bar{x} - Sx < x_{typ} < \bar{x} + Sx$$

$$d = \frac{1}{N}\sum_{i=1}^{N}|x_i - \bar{x}|$$

$$d = \frac{1}{N}\sum_{i=1}^{k}|x_i - \bar{x}|\, n_i$$

$$d = \frac{1}{N}\sum_{i=1}^{k}|\dot{x}_i - \bar{x}|\, n_i$$

$$R = x_{max} - x_{min}$$

$$Q = \frac{Q_3 - Q_1}{2}$$

$$Me - Q < x_{typ} < Me + Q$$

$$Vx = \frac{Sx}{\bar{x}}100\%$$

$$Vd = \frac{d}{\bar{x}}100\%$$

$$V_Q = \frac{Q}{Me}100\%$$

$$V_{Q_1Q_3} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Miary asymetrii

$$As = \frac{\bar{x} - D_0}{Sx}$$

$$Ad = \frac{\bar{x} - D_0}{d}$$

$$A_Q = \frac{Q_3 + Q_1 - 2Me}{2Q} = \frac{(Q_3 - Q_2) - (Q_2 + Q_1)}{(Q_3 - Q_2) + (Q_2 + Q_1)}$$

Analiza korelacji i regresji

Współczynniki korelacji

$$r_{xy} = r_{yx} = \frac{cov(xy)}{SxSy}$$

$$dr_{xy} = r_{xy}^2 \cdot 100\%$$

$$d'r_{xy} = 100\% - dr_{xy}$$

$$cov(xy) = cov(yx) = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})\,(y_i - \bar{y})$$

$$r_s = 1 - \frac{6\sum(kx - ky)^2}{N(N^2 - 1)}$$

$$dr_s = r_s^2 \cdot 100\% \qquad d'r_s = 100\% - dr_s$$

$$\varphi^2 = \frac{ad - bc}{\sqrt{(a + b)(a + c)(b + d)(c + d)}}$$

Równanie regresji liniowej

$$\hat{y} = a_o + a_1 x$$

$$a_1 = \frac{cov(xy)}{S^2x} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} = r_{xy}\frac{Sy}{Sx} \qquad a_0 = \bar{y} - a_1\bar{x}$$

$$S^2u = \frac{1}{n - k}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2 \qquad Su = \sqrt{S^2u}$$

$$R^2 = \frac{\sum(\hat{y}_i - \bar{y})^2}{\sum(y_i - \bar{y})^2} \qquad R^2 = 1 - \varphi^2 = r_{xy}^2$$

Indeksy dynamiki

Przyrosty absolutne i względne

$$\Delta_{t/k} = y_t - y_k \qquad \Delta_{t/t-1} = y_t - y_{t-1}$$

$$P_{t/k} = \frac{\Delta_{t/k}}{y_k} = \frac{y_t - y_k}{y_k} \qquad P_{t/t-1} = \frac{\Delta_{t/t-1}}{y_{t/t-1}} = \frac{y_t - y_{t/t-1}}{y_{t/t-1}}$$

$$i_{n/1} = \frac{y_n}{y_1} = 1 + P_{n/1} \qquad i_{n/n-1} = \frac{y_n}{y_{n-1}} = 1 + P_{n/n-1}$$

$$\bar{\imath}_G = \sqrt[n-1]{i_{n/n-1} \cdot i_{n-1/n-2} \cdot \ldots \cdot i_{2/1}} = \sqrt[n-1]{i_{n/1}} = \sqrt[n-1]{\frac{y_n}{y_1}}$$

$$\bar{T}_n = \bar{\imath}_G - 1 \qquad \bar{T}_n = (\bar{\imath}_G - 1) \cdot 100\%$$

indeksy jednopodstawowe t=1 → indeksy łańcuchowe

$$i_{n/n-1} = \frac{i_{n/1}}{i_{n-1/1}} \qquad \frac{y_n}{y_{n-1}} = \frac{\frac{y_n}{y_1}}{\frac{y_{n-1}}{y_1}}$$

indeksy łańcuchowe → indeksy jednopodstawowe t=1

$$i_{n/1} = i_{n/n-1} \cdot i_{n-1/n-2} \cdot i_{n-2/n-3} \cdot i_{n-3/n-4} \cdot \ldots \cdot \frac{i_2}{i_1}$$

$$\frac{y_n}{y_1} = \frac{y_n}{y_{n-1}} \cdot \frac{y_{n-1}}{y_{n-2}} \cdot \frac{y_{n-2}}{y_{n-3}} \cdot \ldots \cdot \frac{y_2}{y_1}$$

indeksy łańcuchowe → indeksy jednopodstawowe

$$i_{n/k} = i_{n/n-1} \cdot i_{n-1/n-2} \cdot i_{n-2/n-3} \cdot \ldots \cdot i_{k+1/k}$$

$$\frac{y_n}{y_k} = \frac{y_n}{y_{n-1}} \cdot \frac{y_{n-1}}{y_{n-2}} \cdot \frac{y_{n-2}}{y_{n-3}} \cdot \ldots \cdot \frac{y_{n-1}}{y_k}$$

$$i_{k/k} = 1 \qquad i_{n/k} = \left[i_{k/k-1} \cdot i_{k-1/k-2} \cdot \dots \cdot i_{2/1}\right]^{-1}$$

indeksy jednopodstawowe t=1→ indeksy jednopodstawowe o innej podstawie t≠1

$$i_{n/k} = \frac{i_{n/1}}{i_{k/1}}$$

Indywidualne indeksy dynamiki

$$i_p = \frac{p_n}{p_0} \qquad i_q = \frac{q_n}{q_0} \qquad i_w = \frac{w_n}{w_0}$$

$$w_0 = p_0 \cdot q_0 \qquad w_n = p_n \cdot q_n$$

$$i_w = i_p \cdot i_q$$

Agregatowe indeksy dynamiki

$$I_w = \frac{\sum w_1}{\sum w_0} = \frac{\sum p_1 q_1}{\sum p_0 q_0} \qquad P_w = (I_w - 1) \cdot 100\%$$

$$I_p^L = \frac{\sum p_1 q_0}{\sum p_0 q_0} \qquad I_p^P = \frac{\sum p_1 q_1}{\sum p_0 q_1} \qquad I_p^F = \sqrt{I_p^L \cdot I_p^P}$$

$$P_p = \left(I_p^F - 1\right) \cdot 100\% \qquad P_q = \left(I_q^F - 1\right) \cdot 100\%$$

$$I_q^L = \frac{\sum q_1 p_0}{\sum q_0 p_0} \qquad I_q^P = \frac{\sum q_1 p_1}{\sum q_0 p_1} \qquad I_q^F = \sqrt{I_q^L \cdot I_q^P}$$

Równości indeksowe

$$I_w = I_p^L \cdot I_q^P = I_p^P \cdot I_q^L = I_p^F \cdot I_q^F$$

$$\sum q_1 p_0 = \sum p_0 q_0 \cdot \frac{q_1}{q_0} = \sum w_0 \cdot i_q \qquad \sum p_1 q_0 = \sum p_1 q_1 \cdot \frac{q_0}{q_1} = \sum \frac{p_1 q_1}{\frac{q_1}{q_0}} = \sum \frac{w_1}{i_q}$$

$$\sum q_0 p_1 = \sum \frac{p_1}{p_0} \cdot p_0 q_0 = \sum w_0 \cdot i_p \qquad \sum p_0 q_1 = \sum \frac{p_0}{p_1} \cdot p_1 q_1 = \sum \frac{p_1 q_1}{\frac{p_1}{p_0}} = \sum \frac{w_1}{i_p}$$

$$i_p = 1 + \frac{P_p}{100} = \frac{p_1}{p_0} \qquad i_q = 1 + \frac{P_q}{100} = \frac{q_1}{q_0}$$

$$q_0 = \frac{w_0}{p_0} \qquad q_1 = \frac{w_1}{p_1}$$

Zmienna losowa

$$\int_a^b x^c dx = \left.\frac{x^{c+1}}{c+1}\right|_a^b$$

Zmienna losowa skokowa

$$E(x) = \sum x_i \cdot p_i \qquad E(x^2) = \sum x_i^2 \cdot p_i$$

$$D^2(X) = \sum \left(x_i - E(X)\right)^2 \cdot p_i \qquad D(X) = \sqrt{\sum \left(x_i - E(X)\right)^2 \cdot p_i}$$

$$D^2(X) = \sum \left(x_i - E(X)\right)^2 p_i = \sum x^2 p_i - \left(E(X)\right)^2 = E(x^2) - E^2(X)$$

Zmienna losowa ciągła

$$F(x) = \int_{-\infty}^{\infty} f(t)dt \quad dla\ x \in R$$

$$E(X) = \int_{-\infty}^{\infty} x\, f(x)dx$$

$$D^2(X) = \int_{-\infty}^{\infty} \left(x - E(X)\right)^2 f(x)dx$$

Notatki